THE POSITION OF MATHEMATICS IN THE HIERARCHY OF SPECULATIVE SCIENCE

I. Introduction

1. Statement of the problem

Thomistic interest in the problem of the classification of mathematics is not prompted by any desire to minimize the legitimate progress of modern mathematics. Increasing indications of such an interest are, in great measure, due to a belated recognition of the true character of that progress. Faced with overwhelming evidences of major advances in mathematics and the mathematical sciences since the time of St. Thomas, Thomists have been literally forced to a re-examination of the hitherto unchallenged foundations of classical mathematics. It is this renewed study which has served to eliminate many of the apparent conflicts between traditional and modern concepts of the science and to focus attention on the central issue—the position of mathematics among the speculative sciences.

From the Thomistic viewpoint of a hierarchy of speculative sciences, all the progress made within the field of pure mathematics is real progress, consonant with the principles of St. Thomas. It is in the extension of mathematics to other fields that difficulties arise. The genuine utility of mathematics in interpreting the data of natural science has led to abuses. Not content with a regulative role of its own in the physico-mathematical sciences, mathematics has refused to concede a similar role to metaphysics in the philosophy of nature. It has come to deny any higher order of knowledge than its own, to consider itself self-sufficient, independent of all other sciences and capable of interpreting them fully by its superior principles. The conflict here is unequivocal. The traditional conception of a hierarchy of science, while permitting a limited regulative role to mathematics in interpreting data of the lower sciences,

demands a certain dependence of mathematics upon metaphysics, the supreme regulator of all science. Any attempt to determine the precise limits of these regulated and regulative functions of mathematics must clearly indicate the mutual relations between mathematics and the other speculative sciences; it must fix the position of mathematics in an adequate division of speculative science.

The difficulties inherent in such an endeavor are manifest. Yet, most Thomists agree that the solution to the problem is radically contained in St. Thomas' commentary, *In Librum Boetii de Trinitate.* An analysis of the pertinent sections in this commentary, supplemented by parallel passages from other works of St. Thomas, has been made the basis for a division of speculative sciences which is proposed as applicable to modern developments in mathematics and compatible with Thomistic principles. The whole project has been animated by the spirit suggested by Maritain:

> Particularly in relation to the foundations of mathematics much more preliminary work is still required, in my opinion, before Thomist philosophy can propound a systematic interpretation in which all the critical problems offered by modern developments in the mathematical sciences find a solution.[1]

Admittedly deficient in many respects, the present work is offered as a preliminary step toward the ultimate goal of systematic interpretation set by Maritain.

II. TEXTUAL KEY TO THE PROBLEM

The key texts of St. Thomas on the division of speculative science are found in the fifth and sixth questions of his opusculum, *In Librum Boetii de Trinitate.* Each question consists of four articles, distributed as follows:

Question V: The Division of Speculative Science.

Art. I: Whether speculative science is conveniently divided into natural, mathematical, and divine science.

[1] J. Maritain, *The Degrees of Knowledge* (New York: 1938), p. xiii.

Art. II: Whether natural philosophy is of those things which are in motion and matter.

Art. III: Whether mathematical consideration is without motion and matter.

Art. IV: Whether divine science is of those things which are without matter and motion.

Question VI: The Modes which he attributes to Speculative Science.

Art. I: Whether it is necessary to proceed in natural science *rationabiliter*, in mathematics *disciplinabiliter*, and in divine science *intelligibiliter*.[2]

Art. II: Whether imagination is absolutely relinquished in divine science.

Art. III: Whether our intellect can consider the divine form itself.

Art. IV: Whether this can be by way of some speculative science.

This order of procedure has its origin in the text of Boetius. In his prologue to the fifth question, St. Thomas makes it clear that the division of science under consideration is that which Boetius has placed in his text and that the modes are those which the latter attributes to speculative science. The point is not without significance. St. Thomas, commenting on the text of Boetius, reveals his own doctrine on the division of

[2] Science, in the philosophical sense of the term, is certain and evident knowledge acquired through demonstration. In relation to the teacher, this knowledge is called *doctrine*; in relation to the student, it is called *discipline*. Metaphysics, mathematics, and natural science may therefore be rightfully termed *disciplines*. But St. Thomas maintains that mathematics is most properly said to proceed *disciplinabiliter* since mathematical principles and conclusions are most readily grasped by the student and mathematical demonstrations generate the greatest certitude in the student's mind. For the rigid demonstrations of mathematics proceed solely from the standpoint of formal causality.

Similarly, evidence is presented in support of the view that the *reasoning* process is particularly characteristic of the demonstrations of natural science and that *understanding* is the characteristic feature of metaphysical demonstrations. For in natural science we reason from extrinsic causes, from one thing to another, but in metaphysics we understand the very principles of being itself and, understanding them, we are able not only to gain new knowledge but to deepen our understanding of conclusions attained in the other sciences.

speculative science but the order of presentation and method of development are necessarily conditioned by the original text. The real clue to St. Thomas' own approach to the problem seems to be given in the first article of the fifth question.

Citing the authority of Aristotle and Ptolemy in support of the threefold division into natural, mathematical, and divine science proposed by Boetius, St. Thomas proceeds immediately with a reasoned defense of that division. Two preliminary steps in his exposition offer little difficulty. Consideration of the practical sciences is first eliminated by the familiar distinction between the respective ends of the speculative and practical intellect.[3] As a basis for further distinction between the various speculative sciences, St. Thomas advances the equally familiar doctrine of the specification of habits and potencies by their objects.[4] There can be as many specifically distinct speculative sciences as there are formally different objects of the speculative intellect.

It is in the application of this principle of specification that St. Thomas indicates the direction in which his own division of speculative science would proceed. He points out a twofold aspect, of immateriality and immobility, under which all things qualify as objects of the speculative intellect (*speculabilia*). The immaterial character of the intellectual potency demands a corresponding immateriality in its object.[5] This is consonant with St. Thomas' consistent teaching that immateriality is the root of knowledge. In postulating immobility in the object of the speculative intellect, he argues from the nature of the habit of science by which the intellectual potency is perfected. Science is concerned only with necessaries and the necessary is immobile.[6] Insisting on this dual specifying element in the object

[3] Speculativus intellectus, in hoc proprie ab operativo, sive practico distinguitur, quod speculativus habet pro fine veritatem quam considerat, practicus autem veritatem consideratam ordinat in operationem tanquam in finem (*In lib. Boet. de Trin.*, q. 5, a. 1).

[4] Sciendum tamen, quod quando habitus, vel potentiae penes objecta distinguuntur, non distinguuntur penes quaslibet differentias objectorum, sed penes illas quae sunt per se objectorum in quantum sunt objecta (*ibid.*).

[5] *Summa Theol.*, I, q. 85, a. 1.

[6] Omne autem necessarium in quantum hujusmodi, est immobile: quia omne quod

of speculative science, St. Thomas holds that material things, by their very nature, are adapted to separation from matter and motion (*per se competit separatio et a materia et a motu*) and that the speculative sciences are distinguished according to the order of this remotion from matter and motion (*secundum ordinem remotionis et a materia et a motu*).

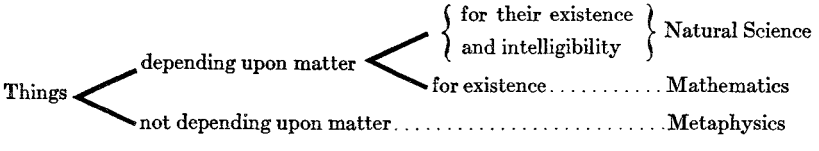
The article concludes with a description, in terms of dependence on matter, of the distinctive objects (*speculabilia*) which are embraced in the three basic genera of speculative science:

(1) Natural Science: includes those things which are utterly dependent upon matter; things which can exist only in matter; things which cannot be understood without matter. Definitions in this order necessarily include sensible matter, as for example, flesh and bones are included in the physical definition of man.

(2) Mathematics: includes those things which have a mixed or partial dependence upon matter; things which exist in matter but can be understood without matter; things which may be defined without reference to matter, as lines and numbers.

(3) Metaphysics: includes those things which are independent of matter; things which not only can be understood and defined without matter but can exist without matter.

A schematic representation reveals the adequate character of this fundamental division:



St. Thomas rejects the possibility of a fourth genus concerned with things depending upon matter for their intelligibility but

movetur in quantum hujusmodi, possibile est esse et non esse vel simpliciter, vel secundum quid (*In lib. Boet. de Trin., loc. cit.*).

not for their existence.[7] The immateriality of the intellect is incompatible with such a hypothesis.

The key to St. Thomas' own division of speculative science is found in this first article in the phrase, "the speculative sciences are distinguished according to the order of remotion from matter and motion."[8] He visualizes the object of speculative science as controlled by two factors: the natural power of the intellect dematerializing its object and the acquired habit of science perfecting this natural knowledge by immobilizing it, by passing judgment on it in terms of necessity. He seems to view the repeated acts of judgment, according to certain necessary and immobile principles, as generating a habit of science which is conditioned by those principles and which, in turn, judges its object in terms of those selfsame necessary and immobile principles. Obviously, any adequate division of speculative science purporting to be Thomistic must take full cognizance of St. Thomas' teaching regarding this remotion from matter by the intellectual potency and from motion by the habit of science.

Preliminary to such a division, two steps were deemed necessary:

(1) A brief restatement of some fundamental notions of abstraction, the doctrinal key to St. Thomas' division of speculative science;

(2) A complete recasting of the doctrine of St. Thomas' *In Librum Boetii de Trinitate*, supplemented by his other works, in an effort to determine the respective roles played by the intellect and by the habit of science in conditioning the objects of the various speculative sciences.

[7] Non est autem possibile, quod sint aliquae res quae secundum intellectum dependeant a materia, et non secundum esse, quia intellectus, quantum est de se, immaterialis est: et ideo non est quartum genus philosophiae praeter praedicta (*ibid.*).

[8] Secundum ordinem remotionis et a materia, et a motu scientiae speculativae distinguuntur (*ibid.*).

III. DOCTRINAL KEY TO THE PROBLEM

The doctrine of abstraction is commonly accepted as the key to the Thomistic division of science.[9] But apart from the innate subtlety of the doctrine, misconceptions are frequently occasioned by new terms and new divisions introduced by later writers in an effort to clarify the position of St. Thomas.[10] Certainly the sense in which he uses the term, abstraction, is not always too clear and that fact offers a real, though not insurmountable, obstacle. It is hoped that a brief summary of the basic notions of abstraction, pertinent to the problem at hand, will facilitate the understanding of applications made by St. Thomas in elaborating his doctrine on speculative science and will permit the retention of his terminology in our own analysis of that doctrine.

The human intellect understands material things by abstraction from phantasms.[11] St. Thomas bases this conclusion on the principle that the object of knowledge is proportionate to the power of knowing. In the cognitive order, man's intellect stands midway between the sensitive powers, which are acts of corporeal organs, and the angelic intellect, which is in no way connected with corporeal matter. It is not the act of an organ nor is it completely independent of matter. It is a power of the immaterial soul which is the form of the human body and the object proportionate to such a power is a form existing individually in matter but not as existing in this individual matter. Only by separating the form from the individual matter which is represented in the phantasm, is the intellect able to know what is in individual matter, though not as existing in such matter. Some such separation of the form from matter or remotion of matter from the form is necessarily entailed in the intellect's attainment of its proper object. St. Thomas terms this process, abstraction, and places it as a necessary condition of human knowledge: " we must needs say that

[9] J. T. Casey, *The Primacy of Metaphysics* (Catholic Univ.: 1936), p. 76.
[10] F. A. Blanche, " L'abstraction," *Bibliotheque Thomiste*, III (1923), 237 ff.
[11] *Summa Theol.*, I, q. 85, a. 1.

our intellect understands material things by abstracting from the phantasms." [12]

Further insight into his conception of the nature of this abstractive process is afforded when he points out, in response to an objection, that abstraction may occur in two ways:

> First, by way of composition and division; thus we may understand that one thing does not exist in another, or that it is separate therefrom. Secondly, by way of simple and absolute consideration; thus we understand one thing without considering the other. Thus for the intellect to abstract one from another things which are not really abstract one from another, does, in the first mode of abstraction, imply falsehood. But, in the second mode of abstraction, for the intellect to abstract things which are not really abstract from one another, does not involve falsehood, as clearly appears in the case of the senses. For if we understood or said that color is not in a colored body, or that it is separate from it, there would be error in this opinion or assertion. But if we consider color and its properties, without reference to the apple which is colored; or if we express in a word what we thus understand, there is no error in such an opinion or assertion, because an apple is not essential to color, and therefore color can be understood independently of the apple.[13]

St. Thomas evidently attributes these two modes of abstraction to the corresponding acts of judgment and simple apprehension. Logical truth, found only in the judgment,[14] is attained through abstraction by way of composition and division; ontological truth, characteristic of the simple apprehension,[15] is attained through abstraction by way of simple and absolute consideration. A critical analysis of the objective basis for these two modes of abstraction and for the truth attained through them is made by St. Thomas in his determination of the objects of the various speculative sciences. The details of that analysis and the conclusions drawn from it, are only intelligible in relation to this primary distinction between the modes of abstraction. Failure to grasp its full significance is fatal to any adequate

[12] *Ibid.*
[13] *Ibid.*, ad 1um.
[14] J. Gredt, *Elementa Philosophiae* (Friburg: 1937), II, 47 ff.
[15] *Ibid.*

comprehension of the respective roles St. Thomas assigns to the intellect and to the habit of science in conditioning the objects of the speculative sciences.

II. Conditioning of the Object in Speculative Science

1. The Role Played by the Intellect

1. *Immateriality as the basis of intellectual knowledge.* In his defense of Boetius' primary division of speculative science, St. Thomas does not rest his case with a demonstration of the mere fitness or convenience of that division. He goes a step further and proposes physics, mathematics, and metaphysics as three basic genera of speculative science. Boetius made no such claim. This view of the three primary divisions of speculative science as basic genera is St. Thomas' own doctrine. Basing his conclusion on the dependence of things upon matter, he excludes all possibility of a fourth genus.[16]

It is not surprising to find St. Thomas' defense of Boetius' division thus resolved in terms of the dependence of things upon matter. Immateriality has traditionally been accepted by Thomists as the basis of all intellectual knowledge and a strict proportion between the intelligibility of a thing and its separability from matter has always been recognized.[17] The entire doctrine of St. Thomas on the speculative sciences is, in fact, an application and a refinement of his fundamental thesis that immateriality is the root of knowledge. The close kinship between his general theory of knowledge and his conception of speculative science is unmistakable. Evidence of that relationship is particularly striking in his analysis of the role played by the intellect in conditioning the objects of the various speculative sciences.

The Thomistic theory of knowledge starts with the fact that we know things. St. Thomas' explanation of that fact may be

[16] Cf. footnote 7, *supra.*

[17] Sicut res sunt separabiles a materia, sic circa intellectum sunt (*Summa Theol., loc. cit.,* sed contra).

summarily stated: our intellect, by the process of abstraction, can possess the forms of material things in an immaterial way.[18] In enumerating three types of things in terms of their separability from matter, St. Thomas is making a more precise determination of the basic fact upon which the whole theory of knowledge depends; he is distinguishing three generic classes among the things that are presented to man's intellect as objects of speculation. His explanation of the way in which the intellect attains these objects may be similarly summarized: the intellect, according to three characteristic types of abstraction, can possess the forms of material things in three degrees of immateriality. The sections which follow are devoted to the clarification and justification of this tentative summation.

2. *Immateriality in the physical order.* The objective basis for abstraction in this order is founded on the composition of the whole with its parts.[19] St. Thomas points out that the whole cannot be abstracted from its formal or specific parts but only from its material parts. The specific parts constitute the very essence of the whole and without them the whole cannot be understood. Our understanding of the nature *man*, for example, necessarily includes its specific parts, rational soul and corporeal body. But the material parts are accidental to the essence of the whole and are not necessary to our understanding of it. The fingers, hands, feet, etc., are not specific parts of man; they are material parts, parts of the individual man from which the intellect can completely abstract and consider only the universal nature, *man*.

It must not be thought that the intellect in thus abstracting from the material parts completely strips the thing of all matter and attains only the substantial form. St. Thomas explains that if this were true:

matter would not be included in the definition of material things. Therefore it must be said otherwise, that matter is twofold, com-

[18] *Summa Theol.*, I, q. 85, a. 1.
[19] *In lib. Boet. de Trin.*, q. 5, a. 3.

mon, and signate or individual; common such as flesh and bone; and individual, as this flesh and these bones. The intellect therefore abstracts the species of a natural thing from the individual sensible matter, but not from the common sensible matter; for example, it abstracts the species of man from this flesh and these bones, which do not belong to species as such but to the individual, and need not be considered in the species: whereas the species of man cannot be abstracted by the intellect from flesh and bones.[20]

Abstraction of this type is termed by St. Thomas, abstraction of the universal from the particular.[21]

Universality is, of course, a necessary condition of all intellectual knowledge since the singular, as such, is not directly attained by the intellect.[22] But in the physical order, St. Thomas attaches a special significance to this abstraction of the universal from the particular. Admitting that the intellect always abstracts from individual sensible matter, he points out that, in the physical order, the abstractive process goes no farther in its remotion from matter.[23] Stripped solely of its individual sensible matter, the material thing attains only the lowest grade of immateriality; it is intelligible only in terms of common sensible matter. Unlike the mathematical and metaphysical orders, the physical order does not exclude common sensible matter from its consideration, for it is concerned only with qualitative being (*ens mobile*), being invested with all the diverse qualities proper to the sensible world;[24] it deals with qualitative changes which are not only manifested through the senses but are also understood only in terms of that common sensible matter in which all sensible qualities reside and have their being.

[20] *Summa Theol., loc. cit.*, ad 2um.

[21] *In lib. Boet. de Trin., loc. cit.*

[22] *Summa Theol.*, I, q. 86, a. 1.

[23] Secundum hoc est sciendum, quod materia signata est individuationis principium, a quo abstrahit omnis intellectus, secundum quod dicitur abstrahere ab hic et nunc. Intellectus autem naturalis non abstrahit a materia sensibili non signata: . . . sed a materia sensibili totaliter abstrahit intellectus mathematicus, non autem a materia intelligibili non signata (*Q. D. de Ver.*, q. 2, a. 6, ad 1um).

[24] J. Maritain, *A Preface to Metaphysics* (London: 1939), p. 81.

3. *Immateriality in the mathematical order.* Abstraction of the universal from the particular, characteristic of the physical order, has an objective basis in the composition of the whole with parts.[25] The universal can be abstracted only from the material parts of the thing and not from the specific parts which constitute its very essence. Abstraction in the physical order is thus limited to a remotion from the individual sensible matter upon which the essence of the thing does not depend. Similiarly, any abstraction based on the composition of matter and form can only be from that matter on which the form does not depend. Accidents, however, are compared to substance as form to matter and it is this composition of accident with substance which St. Thomas proposes as the objective basis for abstraction in the mathematical order.[26]

Since accidents, by their very nature, are dependent upon substance, it is impossible to abstract an accidental form from the substance in which it resides and without which it cannot be understood. But the accidents occur in substance according to a certain order, first quantity, then qualities, and finally passions and motion.[27] Quantity can therefore be understood prior to any consideration of sensible qualities. It can be understood solely in relation to the substance on which it depends.[28]

In the mathematical order, St. Thomas maintains that the intellect strips the thing of all sensible matter, the subject of sensible qualities, leaving only intelligible matter which is nothing more than substance as subject to quantity.

> Mathematical species . . . can be abstracted by the intellect from sensible matter, not only from individual but also from common matter; not from common intelligible matter, but only from indi-

[25] *In lib. Boet. de Trin., loc. cit.*

[26] *Ibid.*; cf. *In II Phys.*, lect. 3.

[27] Similiter autem inter accidentia omnia quae adveniunt substantiae, primo advenit ei quantitas, et deinde qualitates sensibiles et actiones et passiones et motus consequentes sensibiles qualitates (*In II Phys., loc. cit.*).

[28] Unde cum omnia accidentia comparentur ad substantiam sicut forma ad materiam, et cujuslibet accidentis ratio dependeat a substantia, impossibile est aliquam talem formam a substantia separari (*In lib. Boet. de Trin., loc. cit.*).

vidual (intelligible) matter. For sensible matter is corporeal matter as subject to sensible qualities . . . while intelligible matter is substance as subject to quantity. Now it is manifest that quantity is in substance before other sensible qualities are. Hence quantities, such as number, dimension, and figures, which are the terminations of quantity, can be considered apart from sensible qualities; and this is to abstract them from sensible matter; but they cannot be considered without understanding the substance which is subject to quantity; for that would be to abstract them from common intelligible matter. Yet they can be considered apart from this or that substance; for this is to abstract them from individual intelligible matter.[29]

Abstraction of form from matter is thus seen to have a special significance in the mathematical order. It is not to be understood as an abstraction of the substantial form, since the substantial form and the matter corresponding to it are mutually dependent so that one cannot be understood without the other.[30] Nor is it to be understood as abstraction of the accidental form of any sensible quality, since sensible qualities cannot be understood without quantity.[31] It is the abstraction of the accidental form of quantity from sensible matter, individual and common, and from individual intelligible matter. The mathematical order is concerned only with quantitative being (*ens quantum*), being divested of all its sensible qualities and manifesting itself to us only through its quantitative determinations.[32]

4. *Immateriality in the metaphysical order.* The intellect does not achieve complete remotion from matter in either the physical or mathematical orders. In the physical order, abstrac-

[29] *Summa Theol.*, I, q. 85, a. 1, ad 2um.

[30] Similiter autem cum dicimus formam abstrahi a materia, non intelligitur de forma substantiali: quia forma substantialis et materia sibi correspondens, dependent ad invicem, ut unum sine alio non possit intelligi, eo quod proprius actus in propria materia sit (*In lib. Boet. de Trin., loc. cit.*).

[31] Cum qualitates sensibiles non possint intelligi non praeintellecta quantitate, sicut patet in superficie et colore (*ibid.*).

[32] R. E. Brennan, "The Mansions of Thomistic Philosophy," THE THOMIST, I (1939), 65.

tion is limited to a partial remotion from sensible matter. The intellect strips the thing solely of its individual sensible matter, leaving untouched the common sensible matter which is the subject of qualitative determinations. In the mathematical order, remotion from sensible matter is complete. Mathematics leaves behind the world of sensible qualities and is concerned only with quantity. Yet abstraction in the mathematical order involves but a partial remotion from intelligible matter. For while mathematics abstracts from individual intelligible matter, it cannot abstract from common intelligible matter, from substance as subject to quantity. Remotion from common intelligible matter is proper to metaphysics.

The limitations placed on remotion from matter in the physical and mathematical orders have an objective basis in the two-fold composition in things: of the whole with its parts and of accidents with substance. In the physical order there can be no abstraction from the specific parts upon which the whole is dependent; in the mathematical order, there can be no abstraction from substance upon which quantity is dependent. Since the intellect can never abstract from that matter upon which the essence of the thing depends, St. Thomas insists that common sensible matter is essential to our understanding of things in the physical order and not common intelligible matter is essential in the mathematical order.[33]

In the metaphysical order, however, there is no such dependence of things upon matter. Metaphysics is concerned with those things which are completely independent of matter, things which can exist and can be understood without matter (i. e. the *maxime intelligibilia*).[34] St. Thomas points out two distinct classes of things which realize this complete independence from matter.[35] *Separata* are those things, like God and intellectual substances, whose very nature is incompatible with existence in matter. *Separabilia* are those things, like being and substance, whose nature is not incompatible with existence

[33] *In lib. Boet. de Trin.*, q. 5, a. 3.
[34] *In I Metaph.*, proem. i.
[35] *Ibid.*; cf. *in lib. Boet. de Trin.*, q. 5, a. 4.

in matter even though they can exist without matter; they are said to be separated from matter and motion, " not for this reason that it is of their very nature to be *without* matter and motion, as it is the nature of a donkey to be without reason, but for this reason that it is not of their very nature to be *in* matter." [36] *Separata* can never exist in matter; *separabilia* can exist in matter. Things in either class can exist and can be understood without matter. Yet these things, even though they are in themselves most intelligible (*maxime intelligibilia*), are made known to us only through their effects.[37]

All our knowledge begins with the senses, and the proper object of our intellect is the essence (*quidditas*) of the material thing abstracted from its phantasm.[38] Since there are no phantasms of immaterial things, our intellect can only gain an imperfect knowledge of them by stripping the phantasms of material things of all matter, even that common intelligible matter which is found in the mathematical order.[39] Thus it is only by way of privation or negation that these completely immaterial beings, transcending yet permeating the sensible world in which we are immersed, are made known to our intellects.[40] Evidence of this negative abstraction is afforded in the very terminology employed in metaphysics. Negation is either implied in the term itself, as immaterial, individual, etc., or it is expressed in its definition, as when a point is defined as " that which has no parts." [41] But this negative aspect of abstraction in the metaphysical order should not blind us to the positive

[36] Ens et substantia dicuntur esse separata a materia et motu non propter hoc, quod de ratione eorum sit esse sine materia et motu, sicut de ratione asini est sine ratione esse, sed propter hoc quia de ratione eorum non est in materia et motu esse (*In lib. Boet. de Trin.*, q. 5, a. 4, ad 5um).

[37] Per lumen naturalis rationis pervenire non possumus in ea nisi secundum quod in ea per effectus ducimur (*In lib. Boet. de Trin.*, q. 5, a. 4).

[38] *Summa Theol.*, I, q. 88, a. 3.

[39] *Ibid.*, a. 2.

[40] Omnia quae transcendunt haec sensibilia nota nobis per negationem: sicut de substantiis separatis cognoscimus, quod sunt immateriales et incorporeae, et alia huiusmodi (*In III Anima*, lect. 11).

[41] *Summa Theol.*, I, q. 8. 5, a. 8.

intuition of being which is characteristic of that order.[42] For the proper object of the intellect in the metaphysical order is not a particular kind of being, not qualitative being (*ens mobile*) or quantitative being (*ens quantum*), but being itself (*ens*), in all its transcendant perfection.[43]

5. *Summary.* St. Thomas points out a two-fold aspect, of immateriality and immobility, under which things qualify as objects or speculative science.[44] The task of dematerializing its object, rendering it intelligible, is assigned by him to the natural powers of the intellect.[45] Starting with a de facto division into three basic genera of speculative science in terms of dependence on matter, he makes an exhaustive analysis of the objective basis for abstraction in the physical, mathematical, and metaphysical orders.[46] To each order he assigns a distinctive mode of abstraction, a characteristic remotion from matter, which raises the thing understood to a certain level of intelligibility and places it in a certain grade of being.[47]

Since the raw material (*terminus a quo*) of all human knowledge is the sensible material thing, there is a tendency to unduly emphasize the actual process of dematerialization and to neglect the finished product (*terminus ad quem*), the intelligible object which the intellect attains by a simple apprehension. But it is a mistake to consider the abstractive process solely in its negative aspect, as a gradual denudation of matter from form. Maritain observes that:

[42] J. Maritain, *A Preface to Metaphysics* (London: 1939), p. 86.

[43] Nulla scientia particularis considerat ens universale inquantum hujusmodi, sed solum aliquam partem entis divisam ab aliis; circa quam speculatur per se accidens, sicut scientiae mathematicae aliquod ens speculantur, scilicet ens quantum. Scientia autem communis considerat universale ens secundum quod est ens: ergo non est eadem alicui scientiarum particularium (*In IV Metaph.*, n. 532).

[44] Sic igitur speculabilia quod est objectum speculativae scientiae, per se competit separatio et a materia, et a motu (*In lib. Boet. de Trin.*, q. 5, a. 1).

[45] Speculabilia . . . aliquid competit ex parte potentiae intellectivae, et aliquid ex parte habitus scientiae quo intellectus perficitur. Ex parte quippe intellectus competit ei quod sit immateriale, quia et ipse intellectus est immaterialis (*ibid.*).

[46] *Ibid.*

[47] Prima quidem operatio respicit ipsam naturam rei, secundum quam aliqua res intellecta aliquem gradum in entibus obtinet (*In lib. Boet. de Trin.*, q. 5, a. 3).

The essence of the process of abstraction, the eidetic visualization, is not to remove baser minerals in which the precious metal is embedded but to find the metal. The essence of metaphysical visualization is not to remove first the individualising notes, then sensible qualities and finally the quantity, it is that of which all this is but the indispensable condition, namely, the positive perception, the intuition of being as such.[48]

The positive term (*terminus ad quem*) attained by the intellect in the three degrees of immateriality attributed to the three basic genera of speculative science is not neglected by St. Thomas. The intellect, at the corresponding levels of intelligibility, understands the physical, mathematical, or metaphysical essence without reference to the thing in which it previously existed and from which it has been abstracted. Admitting a distinctive type of abstraction in each of these orders, St. Thomas nevertheless considers them all as subdivisions of abstraction by way of simple and absolute consideration.[49] This mode of abstraction, proper to the simple apprehension, is called by him *the understanding of indivisibles* (*intelligentia indivisibilium*)[50] and in enumerating its objects he indicates three indivisibles, three unities which terminate the abstractive process at the three levels of intelligibility.[51]

[48] J. Maritain, *A Preface to Metaphysics* (London: 1939), p. 87.

[49] Since things proper to the metaphysical order can exist and can be understood without matter, St. Thomas notes that they are more properly said to be 'separated' than abstracted. But the simple apprehension of *ens* in the metaphysical order, should not be confused with the separation of the principles of being preliminary to the act of judgment (cf. *In lib. Boet. de Trin., loc. cit.*). Observe that being presents two aspects. One of these is its aspect as *essence* which corresponds particularly to the first operation of the mind. . . . The other is the aspect existence, the *esse* in the strict sense. . . . It is in the second operation of the mind, in the judgment, by composition and division, that the intellect grasps being, not only from the standpoint of essence but from that of existence itself, actual or possible (J. Maritain, *op. cit.*, 19 ff.).

[50] *In lib. Boet. de Trin.*, q. 5, a. 3.

[51] De ea, quae est intelligentia indivisibilium . . . secundum quod indivisibile dicitur tripliciter, quot videlicet modis dicitur et unum, cujus ratio ex indivisione est. Dicitur enim uno modo *aliquid unum continuitate* . . . dicitur alio modo unum, quando habet *speciem unam* . . . prosequitur de indivisibili tertio modo dicto. Videtur enim unum esse quod est penitus indivisibile (*In III Anima*, lect. 11).

The unity attained by abstraction in the physical order is an indivisible according to species (*indivisibile secundum speciem*); in abstracting the universal from the particular, the intellect understands the species itself which unifies the specific parts of the whole.[52] Even though these parts may be actually divided in the thing, the intellect does not apprehend them according to this accidental division but only as they are unified in the universal by the species.[53] In the physical order, a man, a horse, or an army is understood in its universal nature as a specific unity.[54] It is in this sense of a specific unity, an *indivisibile secundum speciem*, that qualitative being (*ens mobile*) is to be understood as the proper object of the intellect in the physical order and not as a complex aggregate of being and mobility.[55]

In the mathematical order the process of dematerialization terminates at an indivisible according to quantity (*indivisibile secundum quantitatem*); the intellect understands the species of quantity (*ipsa species quantitatis*) which unifies the quantitative parts.[56] Unlike the specific parts in the physical order, these quantitative parts are actually undivided in the thing even though they are potentially divisible.[57] A triangle or a

[52] Et licet illud quod est indivisibile specie, habet aliquam divisionem in partibus, tamen illa divisa intelligit per accidens, non inquantum sunt divisibilia et ex parte ejus quod intelligitur, et ex parte temporis, sed inquantum sunt indivisibilia: quia in partibus divisis etiam in actu, est aliquid indivisibile, scilicet ipsa species quam intellectus indivisibiliter intelligit . . . in hoc modo est aliquid indivisibile, scilicet species, quae facit omnes partes totius esse unum (*ibid.*).

[53] *Ibid.*

[54] *Ibid.*

[55] A. M. Pirotta, *Summa Philosophiae* (Turin: 1936), II, 2.

[56] Potest ergo intellectus intelligere magnitudinis dupliciter. Uno modo, secundum quod est divisibilis in potentia, et sic intelligit lineam numerando partem post partem . . . alio modo secundum quod est indivisibilis in actu, et sic intelligit eam ut unum quid constans ex multis partibus, et sic intelligit eam simul. Et ideo subjungit, quod similiter tempus et longitudo dividitur vel non dividitur intelligendo (*In III Anima*, lect. 11).

[57] Unde et illud, quod est continuum, indivisibile dicitur, inquantum non est divisum actu, licet sit divisibile in potentia. Hoc est ergo quod dicit, quod cum divisibile dupliciter, scilicet actu et potentia, nihil prohibet intellectum intelligere indivisibile cum intelligit aliquid continuum, scilicet longitudinem, quae est indivisi-

circle is actually a quantitative unity and in the simple apprehension of these things, the intellect understands them as such without considering them as potentially divisible into parts. It is in this sense of a quantitative unity (an *indivisibile secundum quantitatem*) that quantitative being (*ens quantum*) is to be understood as constituting the proper object of the intellect in the mathematical order.

The complete dematerialization realized in the metaphysical order presents to the intellect objects which are absolutely indivisible (*penitus indivisibile*).[58] The transcendentals proper to metaphysics are both actually and potentially indivisible; they are known to us only by a negation of the actual division found in the physical order and the potential division found in the mathematical order. St. Thomas gives the example of a point, which is a sign of division between the parts of a line but is itself indivisible and made known to us only through the privation or negation of division.[59] These are the ultimate indivisibles, indivisibles rooted not in the unity of physical or mathematical being but in the unity of being itself. In the metaphysical order the human intellect comes in contact with the immaterial, immutable essences of all reality; in that order is achieved the loftiest and most precious intuition of our intellect, that of being as such.[60]

II. THE ROLE PLAYED BY THE HABIT OF SCIENCE

1. *Immobility as the basis of scientific knowledge.* St. Thomas postulates a twofold aspect, of immateriality and of immobility, under which things qualify as objects of speculative

bilis actu, licet sit divisibilis potentia. Et propter hoc intelligit eam cum indivisibili tempore, quia intelligit eam, ut indivisibile (*ibid.*).

[58] Videtur enim unum esse quod est penitus indivisibile, ut punctus et unitas: et de hoc ostendit nunc, quomodo intelligitur: dicens, quod punctum, quod est quoddam signum divisionis inter partes lineae, et omne quod est divisio inter partes continui, sicut instans inter partes temporis, et sic de aliis, et omne quod est sic indivisibile in potentia et actu, ut punctus, "monstratur," idest manifestatur intellectui "sicut privatio," idest per privationem continui et divisibilis (*ibid.*).

[59] *Ibid.*

[60] J. Maritain, *A Preface to Metaphysics* (London: 1939), p. 86.

science. The task of dematerializing things, rendering them intelligible, is assigned by him to the natural powers of the intellect. The habit of science, perfecting the intellect, further refines this immaterial, intelligible object by immobilizing it, by passing judgment on it in terms of its necessary and immobile principles. For all science is of necessaries and the necessary is immobile.[61]

There are, of course, sciences like physics and chemistry which treat of contingent things, things whose behavior is conditioned by the matter which they contain. Considered in themselves, these things are contingent and as such they are known directly by the senses.[62] The senses report the contingent fact; they supply the observational data which is the raw material for all science. The first refinement of that material is effected through remotion from matter by the natural abstractive power of the intellect. The thing thus stripped of its matter, the source of contingency and motion, is apprehended by the intellect as a necessary and immobile essence. It is upon these necessary and immobile essences or forms abstracted by the intellect that all science is founded; it is through them that knowledge of even contingent and material things is made possible to man's intellect.[63] All science, therefore, is said to be necessary because of its ultimate basis in the necessary essences abstracted from things, even though the things themselves may be either contingent or necessary.[64]

In its first apprehension, the intellect is concerned only with essences; it simply understands the immaterial essence in itself without any reference to its previous mode of existence. But St. Thomas points out that this essence or form may be compared to the thing from which it was abstracted.[65] By a certain

[61] Cf. *supra*, note 5.

[62] *Summa Theol.*, I, q. 86, a. 3.

[63] Per hujusmodi rationes immobiles, et sine materia particulari consideratas, habetur cognitio in scientia naturali de rebus mobilibus et materialibus (*In lib. Boet. de Trin.*, q. 5, a. 2.

[64] *Summa Theol., loc. cit.*

[65] Scientia est de aliquo dupliciter. Uno modo primo et principaliter, et sic

power of reflection, the intellect can use the abstract form as a medium of further knowledge (*ut medio sciendi*).[66] The very forms abstracted by the first act of the mind thus become the principles through which this initial knowledge of essences is perfected; through them scientific knowledge is obtained of all things, even those material and changing things which pertain to natural science.[67]

2. *Immobility in the three basic genera of science.* Corresponding to the three degrees of immateriality at which the intellect apprehends its object, Thomists have traditionally recognized three orders of knowledge, three levels of intelligibility. According to the quiddity or nature apprehended by the intellect in its first operation, the thing understood attains a certain grade in being.[68] In the physical order the nature understood is qualitative being (*ens mobile*); in the mathematical order, quantitative being (*ens quantum*); and in the metaphysical order, being itself (*ens*). These three orders, each with its proper object, are considered by St. Thomas as constituting three basic genera of speculative science, but they are not specific sciences. In its first operation, our intellect does not gain scientific knowledge; it simply apprehends the nature or essence of a thing. This initial understanding of essences is perfected by the habit of science, passing judgment on things as they actually exist.

John of St. Thomas holds that the first operation of the intellect constitutes a thing in the intelligible order (*in esse intelligibili*); it presents an immaterial essence which the intellect understands in itself without reference to the thing from which the essence has been abstracted.[69] He considers the thing

scientia est de universalibus rationibus super quas fundatur. Alio modo est de aliquibus secundario, et quasi per reflexionem quandam, et sic de rebus illis est quarum sunt illae rationes, in quantum rationes illas applicat ad res particulares, quarum sunt adminiculo inferiorum virium. Ratione enim universali utitur sciens et ut re scita, et ut medio sciendi (*In lib. Boet. de Trin.*, q. 5, a. 2, ad 4um).

[66] *Ibid.* [67] Cf. *supra*, note 62. [68] Cf. *supra*, note 46.

[69] Supponit enim D. Thomas pro fundamentali principio, quod unaquaeque res in tantum est intelligibilis, in quantum est a materia separabilis, eo quod intelligibile

thus existing—existing, that is, as an object of the intellectual potency (*in esse intelligibili*)—to be the proximate foundation of scientific knowledge. "*Esse scibile* adds over and above the *esse intelligibile* such a mode of knowing, namely, that something is known not simply but illatively, proceeding from causes or premises to conclusions." [70] Since the root of intelligibility is immateriality and we distinguish differences in intelligibility (*intelligibilitas*) according to corresponding differences in immateriality, he reasons that we should similarly distinguish differences in knowability (*scibilitas*) according to differences in immateriality, not considered absolutely in itself but as proceeding from principles to conclusions. Correlating his terminology with that of St. Thomas, his view seems to be that the degree of immateriality realized in the essence of the thing constitutes it in the intelligible order (*in esse intelligibili*) and that the corresponding degree of immobility realized in the principles of that essence constitutes the thing in the scientific order (*in esse scibili*).

This interpretation appears to be consistent with the fundamental principle of St. Thomas that the specification of the object in speculative science is controlled by two factors: immateriality, demanded by the nature of the intellect; and immobility, demanded by the habit of science. For St. Thomas maintains that "as each science considers a certain genus, it is

idem est quod spirituale et immateriale, principium autem spiritualitatis est denudatio a materia. Et sic cum materia obumbret et impediat intelligibilitatem, illuminatur et apparet objectum, secundum quod a materia secernitur diversimode. Cui fundamento addendum est, quod esse scibile addit supra esse intelligibile talem modum cognoscendi, scilicet quod aliquid intelligatur non simplici modo, sed modo illativo, ex causis seu praemissis procedendo ad conclusiones; scire enim est cognoscere causam, obquam res est, etc. *Ex quo manifeste deducitur*, quod si radix intelligibilitatis est immaterialitas, et consequenter diversae intelligibilitatis diversa immaterialitas, ita similiter radix et principium diversae scibilitatis erit diversa immaterialitatis seu abstractio, non absolute et simplici modo considerata, sed prout ex praemissis descendit ad conclusiones. Praemissae autem seu media probativa in scientiis sunt ipsa prima principia seu definitiones, per quas proprie passiones de subjecto demonstrantur (Joannis a Sancto Thoma, *Cursus Philosophicus Thomisticus*, Log. II, q. 27, a. 1).

[70] *Ibid.*

necessary that it consider the principles of that genus, since science is not perfected except through knowledge of first principles." [71] These first principles are concomitant with the different orders of knowledge; [72] they are naturally grasped in our first apprehension of the physical, mathematical, or metaphysical nature of a thing.[73] The difficulty lies in determining just what degree of immobility or necessity it to be attributed to the first principles proper to each of these three orders. For while the three degrees of immateriality are quite clearly indicated in the writings of St. Thomas, a comparable determination of the degrees of immobility is lacking. Yet sufficient basis for the following sections is afforded.

a) *Immobility in the physical order*: In this order, the object presented to the intellect (*in esse intelligibili*) is qualitative being (*ens mobile*). The immateriality achieved at this lowest level of intelligibility is not absolute; common sensible matter constitutes a specific part of every qualitative being and cannot be eliminated in our intellectual apprehension of it. The physical order is concerned with those things which can neither exist nor be understood without common sensible matter; things which contain within themselves the principles of motion.[74] It is through these principles of motion that knowledge is had in the natural sciences.[75] The intellect, however, in using these principles of motion as the means for demonstrating conclusions, is no longer considering qualitative being (*ens mobile*) as

[71] Sciendum igitur quod quaecumque scientia considerat aliquod genus subjectum, oportet quod consideret principia illius generis, cum scientia non perficiatur nisi per cognitionem principiorum primorum (*In lib. Boet. de Trin.*, q. 5, a. 4).

[72] Patet ergo quod ad diversificandum scientias sufficit diversitas principiorum, quam comitatur diversitas generis scibilis (*In Anal. Post.*, lib. 1, lect. 41).

[73] In intellectu nostro sint quaedam quae intellectus noster naturaliter cognoscit, scilicet prima principia, quamvis etiam ipsa cognitio in nobis non determinetur nisi per acceptionem a sensibus (*Q. D. de Ver.*, q. 8, a. 15).

[74] Scientia naturalis est circa ea quae habent in seipsis principium motus (*In XI Metaph.*, lect. 7).

[75] Quia tamen corpus mathematicum cognoscitur per principia quantitatis, corpus autem naturale per principia motus, non est eadem scientia geometria et naturalis (*In Anal. Post.*, lib. 1, lect. 41).

an object of the intellectual potency (*in esse intelligibili*); it is considering *ens mobile ut mobile*, as an object of the habit of science (*in esse scibili*). For natural science considers sensible substances only as they are in act and in motion.[76]

b) *Immobility in the mathematical order*: In this order, the object presented to the intellect (*in esse intelligibili*) is quantitative being (*ens quantum*).[77] The essences with which this order is concerned are abstracted from all sensible matter but not from intelligibile matter.[78] Thus the immateriality attained at the mathematical level of intelligibility transcends the consideration of sensible matter but necessarily involves the consideration of intelligibile matter without which quantitative being is unintelligible. At this level of intelligibility the intellect, by reflecting upon the essence apprehended in its first operation, is able to distinguish principles of quantity which can be used in demonstrating conclusions in the science of mathematics.[79]

If these principles of quantity are expressed in terms of immobility, the difference between them and the principles of motion proper to the physical order becomes clearly apparent. The physical order, concerned with essences which actually contain within themselves the principles of motion, considers its object *in esse scibili* as moving (*ut mobile*); the mathematical order, concerned with essences which are only potentially capable of change, considers its proper object *in esse scibili* as capable of change (*ut mutabile*). For the mathematical order considers quantitative being, *in esse intelligibili*, as a unity which is actually undivided (*indivisibilis in actu*) but the science of mathematics considers it, *in esse scibili*, as potentially divisible (*divisibilis in potentia*).[80] The child who

[76] Naturalis scientia considerat solum de substantiis sensibilibus inquantum sunt in actu et in motu (*In XII Metaph.*, lect. 2).

[77] Cf. note 42, *supra*.

[78] *In VIII Metaph.*, lect. 5.

[79] *In Anal. Post., loc. cit.*

[80] *Summa Theol.*, I, q. 85, a. 8; cf. footnotes 55 and 56, *supra*.

apprehends a line, a circle, and a triangle as distinct unities, understands them at the mathematical level. But scientific knowledge is not attained until he learns to distinguish the quantitative parts and to use them in demonstrating characteristic properties of the different mathematical figures.

c) *Immobility in the metaphysical order*: The object which the intellectual potency attains in this order is being (*ens*). Since the metaphysical order is concerned with those things which can exist and can be understood without matter, the immateriality realized at this level of intelligibility is absolute. Consequently the absolute immateriality of the object *in esse intelligibili* is paralleled by an absolute immutability in the principles which constitute the object *in esse scibili*. For the intellect, reflecting on the essence of being, can distinguish no other principles than the immutable principles of being itself. It is for this reason that the proper object of the science of metaphysics is commonly said to be being as such (*ens ut sic*) or being considered as being (*ens ut ens*). But the proper object, *in esse scibili*, of the science of metaphysics is expressed in terms of its immobility as *ens ut immutabile*. For *ens*, in the metaphysical order, is indivisible in potency and act, and cannot be considered as mobile or mutable.[81]

3. *Summary*. Immobility is posited by St. Thomas as the second requisite for an object of speculative science. The material thing must be stripped not only of matter but of motion before it is fully constituted as an object of science. The natural abstractive power of the intellect dematerializes things and permits their apprehension, *in esse intelligibili*, at three levels of intelligibility, in three degrees of immateriality. Reflecting upon the objects presented *in esse intelligibili*, the intellect can distinguish three corresponding degrees of immobility in their principles and the objects are constituted *in esse scibili* according to these degrees of immobility. But since judgment, the second act of the mind, uses these principles in

[81] *Ibid.*

establishing the conclusions of science, it is imperative that we consider the thing to which these principles are applied, the *terminus ad quem* of judgment. For the principles of the higher orders are applicable also to objects in the lower orders.[82]

The principles of being, proper to the metaphysical order, can be applied not only to things in that order but to those in the two lower orders. From his supreme vantage point at the highest level of intelligibility, the metaphysician is in a position to judge all things in terms of their immutable principles. He can consider *being* as immutable (ens ut immutabile) in the science of metaphysics; *quantitative being* as immutable (ens quantum ut immutabile) in the philosophy of mathematics; and *qualitative being* as immutable (ens mobile ut immutabile) in the philosophy of nature. At the second level of intelligibility, the mathematician is in a position to judge of things in his own and in the physical order in terms of their mutability. He can consider *quantitative being* as mutable (ens quantum ut mutabile) in the science of mathematics and *qualitative being* as mutable (ens mobile ut mutabile) in the physico-mathematical sciences. But at the lowest level of intelligibility, the principles of motion proper to the physical order can be applied only to things in that order; the consideration of qualitative being as mobile or changing (ens mobile ut mobile) is the solitary orbit of natural science.

A division of speculative science based on this two-fold specification of object in terms of immateriality and immobility can now be proposed.

III. Division of Speculative Science

The division of speculative sciences can be most graphically represented by a diagram (see p. 495). Before examining this diagram, however it will be well to recall the main points of the discussion. In our introductory section, the solution of the prob-

[82] Quanto scientia aliqua abstractiora et simpliciora considerat, tanto ejus principia sunt magis applicibilia aliis scientiis (*In lib. Boet. de Trin.*, q. 5, a. 3, ad 6um).

lems raised by modern mathematics was reduced to the determination of the position occupied by mathematics in an adequate division of speculative science, the adequacy of such a division to be judged by: (A) its compatibility with Thomistic doctrine; (B) its applicability to modern developments in mathematics. It is under these two aspects that the merits and advantages of our proposed divisions are discussed.

I. CORRELATION OF DIVISION WITH THOMISTIC DOCTRINE

In seeking a Thomistic approach to an adequate division of speculative science, St. Thomas' commentary, *In Librum Boetii de Trinitate,* was utilized as the key text and his theory of abstraction, as the key doctrine. Radical justification for the division proposed, is found in the analysis of that text and that doctrine made in the preceding section. But additional evidence of its compatibility with St. Thomas' doctrine on the speculative sciences and with his theory of abstraction is indicated in the following sections.

1. *Relation to key texts of St. Thomas.* The speculative sciences are necessarily divided according to differences in the things which are presented to the intellect as objects of science. In St. Thomas' view, these differences in the objects of science (*speculabilia*) are controlled by two factors: immateriality, demanded by the intellectual faculty and immobility, by the habit of science. Our analysis of those two factors, leading to the distinction, not only of three degrees of immateriality, but of three corresponding degrees of immobility seems sufficiently justified in the preceding section. But the interrelation of the degrees of immateriality and immobility in specifying the object of science was merely indicated in the statement that "the principles of the higher orders are applicable also to the lower orders." [83]

a) *In Librum Boetii de Trinitate*: St. Thomas points out that the principles of mathematics are applicable to things in

[83] Oportet scientias speculativas dividi per differentias speculabilium, in quantum sunt speculabilia (*In lib. Boet. de Trin.*, q. 5, a. 1).

the physical order and that an intermediary science (*scientia media*) between purely mathematical and purely natural science is constituted by such application.[84] This intermediary science is considered by him to constitute a specific science, distinct from mathematics and natural science. Pure natural science considers the properties of natural things as such and pure mathematics considers quantity in itself.[85] But the intermediary science (*scientia media*) is formally mathematical and materially physical; its conclusions about things in the physical order are demonstrated through principles of the mathematical order.[86] Astrology and music are cited by St. Thomas as sciences in which such an application of mathematical principles to physical things is made.[87]

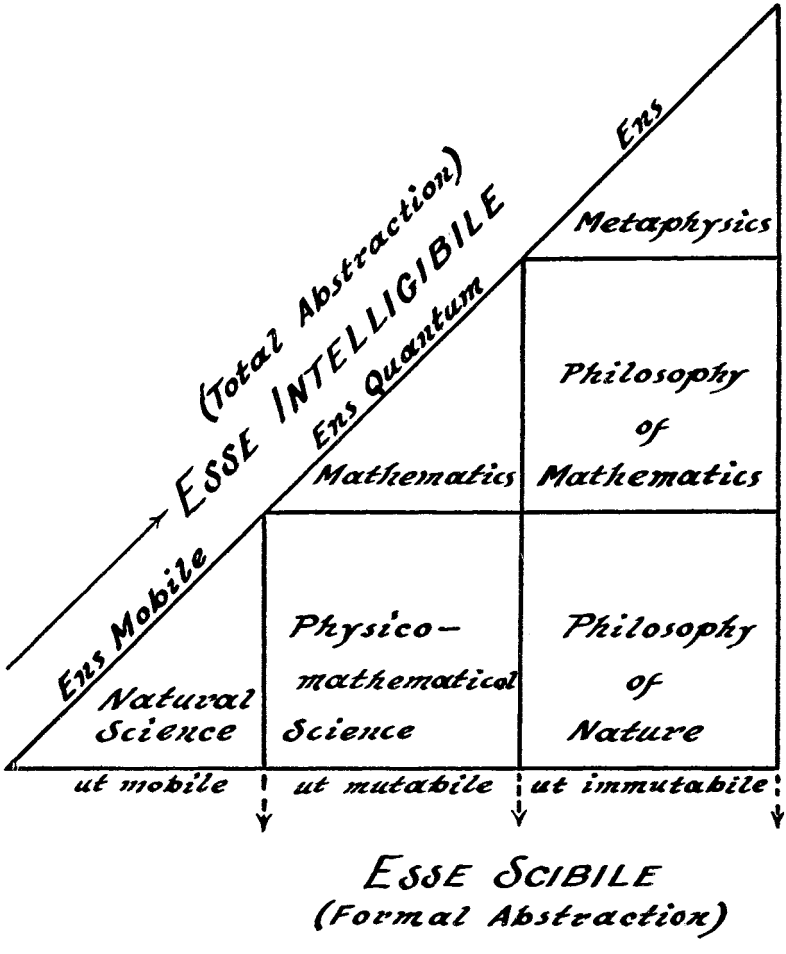
Following the terminology of Maritain,[88] this type of science is indicated in our diagram of the sciences as physico-mathematical. In terms of that diagram, it is the science of *qualitative being as mutable* (*ens mobile ut mutabile*) as distinguished from mathematics, *quantitative being* as mutable (*ens quantum ut mutabile*) and from natural science, qualitative being *as changing* (*ens mobile ut mobile*). Since the principles of the lower order cannot be applied to a higher order, there can be no science of quantitative being as changing (*ens quantum ut mobile*), no science which is formally physical and materially mathematical. There are only three basic sciences possible to the intellect which has not risen above the mathematical order for the principles used in its demonstrations. The other three basic sciences, metaphysics, the philosophy of mathematics,

[84] Et inde est, quod de rebus naturalibus et mathematicis ordines scientiarum tres inveniuntur. Quaedam enim sunt pure naturales, quae considerant proprietates rerum naturalium in quantum huiusmodi, sicut physica, et agricultura, et huiusmodi. Quaedam vero sunt pure mathematicae, quae determinant de quantitatibus absolute, ut geometria de magnitudine, arithmetica de numero. Quaedam vero sunt mediae, quae principia mathematica ad res naturales applicant, ut musica et astrologia, quae tamen magis sunt affines mathematicis, quia in eorum consideratione id quod est physici est quasi naturale: quod autem mathematici, quasi formale (*In lib. Boet. de Trin.*, q. 5, a. 3, ad 6um).

[85] *Ibid.* [86] *Ibid.* [87] *Ibid.*

[88] J. Maritain, *The Degrees of Knowledge* (New York: 1938), p. 52.

DIAGRAM OF SPECULATIVE SCIENCE



and the philosophy of nature are all formally metaphysical, proceeding from the immutable principles of being.

This is indicated in our diagram by their position in the same vertical plane *in esse scibili*, just as their material diversity is indicated by their different horizontal positions *in esse intelligibili*. The immutable principles of being are applied to being itself in the science of metaphysics but to quantitative being in the philosophy of mathematics and to qualitative being in the philosophy of nature. The science of metaphysics alone is the pure science of being, formally and materially metaphysical both in its principles and in the term of its judgments. It is the science of *ens ut immutabile*. But the philosophy of mathematics is formally metaphysical and materially mathematical; it judges the quantified beings of the mathematical order in the light of the immutable principles of the higher metaphysical order. Similarly, the philosophy of nature, in using those same immutable principles to judge the changing things of the physical order, is formally metaphysical but materially physical. For the philosophy of mathematics is the science of *quantitative being* as immutable (*ens quantum ut immutabile*) and the philosophy of nature is the science of *qualitative being* as immutable (*ens mobile ut immutabile*).

Of the six sciences included in our division of speculative science, three are pure sciences and three are intermediary sciences (*scientiae mediae*). Metaphysics, mathematics, and natural science are pure sciences, considering things in the metaphysical, mathematical, and physical orders in the light of the principles proper to each of those orders. Physico-mathematical science is the intermediary science (*scientia media*) between the mathematical and physical orders; the philosophy of mathematics and the philosophy of nature are intermediary sciences (*scientiae mediae*) between the metaphysical and, respectively, the mathematical and physical orders. Thus the six sciences included in our diagram are considered as six basic classes into which all speculative science naturally falls. Further evidence in support of this view is afforded in the following sections.

b) In his *Expositio in Libros Posteriorum Analyticorum:* St. Thomas states that the unity of a science is determined by the unity of its subject, the thing about which judgment is made.[89] He distinguishes three generic subjects of science according to the three degrees of immateriality, giving *ens, corpus mathematicum,* and *corpus naturale* or *mobile* as examples.[90] But to each genus he attributes first principles which are concomitant with it and he maintains that application of the first principles proper to a given order constitutes pure science in that order.[91] Similarly, he gives examples of intermediary sciences (*scientiae mediae*) in which the principles of a higher order are applied to things in a lower order.[92] For he maintains that the sciences are distinguished according to the diversity of their principles.[93]

Evidently the unity of subject which St. Thomas attributes to a science is only a material unity; unity of principles constitutes a formal unity.[94] According to our diagram, such a material unity would be attributed to natural science, physico-mathematical science, and the philosophy of nature, since

[89] Dicit ergo primo quod scientia dicitur una, ex hoc quod est unius generis subjecti. Cuius ratio est, quia processus scientiae cuiuslibet est quasi quidam motus rationis. Cuiuslibet autem motus unitas ex termino principaliter consideratur . . . et oportet quod unitas scientiae consideretur ex fine sive ab termino scientiae. Est autem cuiuslibet scientiae finis sive terminus, genus circa quod est scientia: quia in speculativis scientiis nihil aliud quaeritur quam cognitio generis subjecti (*In Anal. Post.*, lib. 1, lect. 41).

[90] *Ibid.*

[91] Patet ergo quod ad diversificandum scientias sufficit diversitas principiorum, quam comitatur diversitas generis scibilis. Ad hoc autem quod sit una scientia simpliciter utrumque requiritur et unitas subjecti et unitas principiorum (*ibid.*).

[92] Unde patet quod quando ea, quae sunt lineae simpliciter, applicantur ad lineam visualem, fit quodammodo descensus in aliud genus . . . quod est unius scientiae non habet probare alia scientia, nisi forte una scientia sit sub altera, sicut se habet perspectiva ad geometriam, et consonantia vel harmonia, idest musica, ad arithmeticam (*ibid.*, lect. 15).

[93] Cf. note 90, *supra.*

[94] Cum ergo scibile sit proprium objectum scientiae, non diversificabuntur scientiae secundum diversitatem materialem scibilium, sed secundum diversitatem eorum formalem. Sicut autem formalis ratio visibilis sumitur ex lumine, per quod color videtur, ita formalis ratio scibilis accipitur secundum principia, ex quibus aliquid scitur (*In Anal. Post.*, lib. 1, lect. 41).

qualitative being (*ens mobile*) is their common subject-matter. Formally they are distinct sciences, each demonstrating its conclusions through principles proper to different orders. This diversity of principles is indicated diagrammatically in the different positions which each of these sciences occupies *in esse scibili.*

2. *Relation to doctrine of abstraction.* In our introductory section, abstraction was proposed as the doctrinal key to the Thomistic division on speculative science. We signified our intention of following the terminology of St. Thomas, and described his distinction between the modes of abstraction characteristic of the simple apprehension and the judgment as vital to a proper understanding of his doctrine on the division of speculative science. In our analysis of the part played by the intellect and by the habit of science in conditioning the object of science, three sub-divisions of each of these primary modes of abstraction were indicated. The intellect was found to apprehend things in three degrees of immateriality; the habit of science was found to judge things according to principles expressed in three degrees of immobility.

Throughout this analysis, the terminology of St. Thomas was preserved in the interests of accuracy of interpretation and uniformity of expression. It was only in determining the interrelation of these two factors of immateriality and immobility in controlling the object of science, that John of St. Thomas' distinction between *esse intelligibile* and *esse scibile* was introduced as adaptable for a summation of St. Thomas' doctrine. In our diagram, *esse intelligibile* thus expresses the degree of immateriality found in the object abstracted by the first operation of the intellect. It indicates the level of intelligibility achieved in the simple apprehension. *Esse scibile,* indicating the degree of immobility found in the principles used in judgment, completes the expression of the object of science in terms of its immateriality and immobility.

This manner of indicating the distinctions between the various speculative sciences seems perfectly satisfactory and in

harmony with St. Thomas' notion of abstraction as explained in the preceding chapter. But since Thomists, following Cajetan, have traditionally used the terms *total* and *formal* abstraction, a possible application of those terms to our division is suggested parenthetically. The four-fold difference which Cajetan notes [95] between these two modes of abstraction, seems to warrant the view that total abstraction can be attributed to the intellect in its simple apprehension and formal abstraction to the habit of science in its judgments. Cajetan maintains that it is within the different modes of formal abstraction that the speculative sciences are distinguished; total abstraction is common to all science.[96] It must not be thought that he identifies formal abstraction with the intellectual apprehension of quantitative being, nor total abstraction with intellectual apprehension at the physical level.

II. THE POSITION OF MATHEMATICS IN SPECULATIVE SCIENCE

The central issue raised by modern progress in mathematics concerns the position of mathematics with respect to the other speculative sciences. Admitting legitimate progress within the pure science of mathematics and tremendous developments in the physico-mathematical sciences, Thomists have refused to concur in the tacit denial of metaphysics and the substitution of mathematics as the supreme regulator of science.[97] They have been unwilling to abandon the traditional concept of a hierarchy of science which permits a limited regulative role to mathematics in interpretating the data of physical science, yet demands a certain dependence of mathematics on metaphysics. Our diagram of the speculative sciences is offered as affording a more precise indication of these regulated and regulative functions of mathematics, through fixing its position in the hierarchy of speculative science.

[95] Cajetan, *Comm. in de Ente et Essentia*, q. 1.

[96] Quia penes diversos modos abstractionis formalis scientiae speculativae diversificantur . . . . Abstractio autem totalis communis est omni scientiae (*ibid.*). Cf. *supra* (9).

[97] J. Maritain, *op. cit.*, p. 245.

1. *Regulated functions of mathematics.*

a) *Pure Mathematics*: Mathematics, as indicated in our diagram, is not directly regulated by metaphysics. It is pure science at the mathematical level of intelligibility, capable of its own intrinsic development.[98] As the science of quantitative being as mutable (*ens quantum ut mutabile*), it considers quantitative being as quantified, as potentially divisible into quantitative parts which can be numbered or measured.[99] Reflecting on the quantitative being attained by simple apprehension at the mathematical level, the mathematician can distinguish its quantitative parts and use them as principles of demonstration. These quantitative principles are not borrowed from metaphysics. They are principles proper to the mathematical order, concomitant with the mathematical being apprehended in that order.

It is in this sense that mathematics is recognized by Thomists as autonomous within its own field,[100] but that very autonomy is guaranteed and defended by metaphysics. Metaphysics rules mathematics negatively by restricting it to its own proper sphere; by limiting its conclusions to the field of mathematics without dictating those conclusion.[101] Metaphysics, from its position at the summit of human science, restricts mathematics to the lower mathematical order of quantitative being and limits its conclusions to the principles of mutability proper to that order. Reserving to itself the consideration of being as immutable (*ens ut immutabile*), it concedes to mathematics the consideration of quantitative being as mutable (*ens quantum ut mutabile*). But if the mathematician seeks a deeper insight into the quantified beings which he studies, if he wishes to

[98] *Ibid.*, p. 60.

[99] *Ponit species quantitatis;* inter quas primae sunt duae; scilicet multitudo sive pluralitas, et magnitudo sive mensura. Utrumque autem eorum habet rationem quanti, inquantum multitudo numerabilis est et magnitudo est mensurabilis. . . . Multitudo est, quod est divisibile secundum potentiam in partes non continuas. Magnitudo autem quod est divisibile in partes continuas (*In V Metaph.*, lect. 15).

[100] M. J. Adler, *What Man Has Made Of Man* (New York: 1937), p. 27.

[101] *Ibid.*

defend and explain the principles which he uses in his own field, he must look to metaphysics.[102] The ultimate explanation of the axioms and assumptions of all science is rooted in the immutable principles of being itself.[103]

b) *The Philosophy of Mathematics*: This science, as indicated in our diagram, is directly regulated by mathematics. It is the intermediary science (*scientia media*) between mathematics and metaphysics: materially mathematical in its subject-matter, quantitative being (*ens quantum*), and formally metaphysical in its immutable principles. It is the science of quantitative being as immutable (*ens quantum ut immutabile*), a distinct science which uses the immutable principles of metaphysics to gain a deeper knowledge of mathematical being. Only in such a philosophy of mathematics, using the immutable principles of being itself, will the axioms and assumptions of mathematics receive adequate defense and explanation. For while the pure science of mathematics is not directly dependent upon metaphysics for the attainment and use of its first principles, it is utterly incapable of explaining and defending them.[104]

> But metaphysics, from the very fact that it has being as its object, also covers somehow all the inferior sciences which treat of particular beings or particular aspects of beings, and may judge them, defend these sciences and use them. . . . Reflecting on mathematics, metaphysics becomes philosophy of mathematics, which is only materially mathematical, even though the data used be formally mathematical.[105]

2. *Regulative functions of mathematics.*

a) *Physico-mathematical Science*: The regulative role of mathematics in interpreting the data of physical science is indicated, in our diagram, by the position of mathematics

[102] J. Gredt, *op. cit.*, I, 185.

[103] J. Maritain, *op. cit.*, p. 62.

[104] *Ibid.*, p. 60.

[105] C. D. De Koninck, "Thomism and Scientific Indeterminism," *Proc. Am. Cath. Phil. Assoc.*, XII (1936), 75.

immediately over physico-mathematical science. The later science is the intermediary science between the mathematical and physical orders, judging the changing things of the inferior order by the principles of the higher, mathematical order. For although the consideration of motion does not pertain to pure mathematics, St. Thomas cites instances in which the mathematical principles of measure and number are legitimately and effectively employed in studying motion and change in physical things.[106] Maritain, therefore, aptly describes this type of science as

> a *scientia media*, of which the typical examples to the ancients were geometrical optics and astronomy: an intermediary science, halfway between mathematics and empirical natural science, of which the physically real forms the subject-matter in regard to the measurements which it allows us to draw from it, but whose formal object and conceptual procedure remain mathematical: a science which we may call *materially physical* and *formally mathematical*.[107]

In exercising this regulative function, mathematics is perfectly within its rights. As a superior science, it can exert a directive and interpretative influence in physico-mathematical science, comparable to that of metaphysics in the philosophy of mathematics and in the philosophy of nature. But it is imperative that such a comparison take full cognizance of the wide divergence in the principles which mathematics and metaphysics bring to bear on the sciences inferior to them. It must be remembered that physico-mathematical science producing "both scientific knowledge and a marvelous technical power over nature," is proceeding, "from the point of view of quantity, not that of being."[108] In the terms used in our division of science, it is the science of qualitative being as mutable (*ens mobile ut mutabile*). As such it constitutes a specific science, distinct not only from mathematics but also from natural science and the philosophy of nature. Its relations to mathe-

[106] *In lib. Boet. de Trin.*, q. 5, a. 3, ad 5um et 6um.
[107] J. Maritain, *op. cit.*, p. 52.
[108] *Ibid.*, p. 56.

matics, upon which it is formally dependent for its principles, has already been described. The limitations of its field, in relation to the other two basic sciences in the physical order, are indicated in the final section.

b) *Inter-relation of basic sciences in the physical order*: The material identity and formal diversity of natural science, physico-mathematical science, and the philosophy of nature is evident from their relative positions in our diagram of speculative science. Qualitative being (*ens mobile*) is their common subject-matter; their judgments all terminate in the physical order, but the principles each employs are formally distinct, originating in different orders. It is this formal difference in principles which constitutes them specific sciences: natural science, using principles proper to the physical order; physico-mathematical science, principles proper to the mathematical order; and the philosophy of nature, principles of the metaphysical order. Natural science is pure science in the physical order; physico-mathematical science is the intermediary science between the physical and mathematical orders; the philosophy of nature is the intermediary science between the physical and metaphysical orders.

Natural science, pure science at the physical level, considers changing things as changing; it considers qualitative being as mobile (*ens mobile ut mobile*). As indicated by its position, it is not directly dependent upon either mathematics or metaphysics; it is perfectly autonomous within its own field. Yet its very position at the opposite extreme to metaphysics, at the lowest point in *esse intelligibili* and in *esse scibili*, marks the narrow limits of that field. The degree of immateriality realized in its object is the bare minimum sufficient to place it in the intelligible order and the immobility attributed to its principles is the minimum required to constitute it in *esse scibili*. Natural science must, in fact, consider the changing thing as changing (*ut mobile*). It is only by restricting its consideration to a certain type of change, by controlling and correlating all the variant factors, and by multiplying its observations that natural science

achieves a degree of immobility in its principles and a measure of necessity in its conclusions. For natural science seeks only "to discover the interconnexion of the phenomena of change, the correlation of the accidental variables which, in their covariancy, present to us the uniform face of a changing world." [109]

But the mind of man has never rested content with this superficial knowledge gained in natural science. Consciously or not, men have always sought a more radical explanation for the manifold changes of this changing world. The natural attraction of the intellect toward the higher orders has constantly led to the interpretation of the data of natural science in the light of mathematical and metaphysical principles. Evidence of that attraction is discernible at every period in the history of human thought. Its direction and intensity alone have varied. Predominantly mathematical in its trend today, natural science in the past looked almost exclusively to metaphysics for an explanation of problems suggested, but unanswered, in its own restricted field. Failure to recognize these tendencies and their influence on the findings of what is loosely called "natural science," in the generic sense of science at the physical level, has been an endless source of misunderstanding for all concerned.

Adler observes:

> The composite character of ancient physical writings—partly philosophical in method and partly scientific—requires great care in the modern reader, who must distinguish philosophical principles from scientific knowledge of particular matters of fact. In the latter sphere it is to be expected that the ancients would make many errors; but these errors do not affect the validity of their principle.[110]

Following his own suggestion, he classes the *Physics* of Aristotle as "purely philosophical" and the *De Coelo* and the *De Generatione et Corruptione* as "in some part also," philosophical.[111] St. Thomas, in fact, seems to indicate that these

[109] M. J. Adler, *op. cit.*, note 9, p. 147. [110] *Ibid.*, p. 21. [111] *Ibid.*, 19 ff.

three books of Aristotle's works on "natural science" might well be considered in our classification as, respectively, philosophy of nature, physico-mathematical, and natural science.[112] Certainly St. Thomas was not unaware of the essential diversity of these three basic sciences. His insistence upon astronomy as an intermediary science between pure science in the physical and mathematical orders is proof of that.[113]

It is equally important that we recognize the place these three sciences hold in current scientific works. Modern physics, physical chemistry, and to a great extent the whole field of chemistry, are physico-mathematical sciences. Proceeding formally from mathematical principles, they are the modern counterpart of the more ancient intermediary sciences such as astronomy and are included in our diagram under the physico-mathematical sciences. Sciences of the biological type, less amenable to mathematization, continue to consider "sensible and observable being in the very degree to which it is sensible and observable."[114] It is for this reason that they are classed as natural science in our division. However these pure sciences

[112] Praecedunt autem hunc librum, secundum ordinem, in scientia naturali tres libri. Unde tria facit. Primo ponit de quo sit actum in libro *Physicorum*. In quo quidem, quantum ad duos primos libros ejus, agitur de causis naturae, . . . ut intelligantur *primae causae naturae* prima principia, quae sunt materia, forma, et privatio, et etiam quattuor genera causarum, scilicet, materia, forma, agens, et finis. In sequentibus autem libris *Physicorum* agitur de motu in generali . . . Secundus scientiae naturalis liber est liber *de Coelo et Mundo*, . . . Tertius liber scientiae naturalis est liber *de Generatione*: in quo determinat de permutatione elementorum in invicem, in secundo libro, et de generatione et corruptione in communi in primo libro (*In I Meteorlogicorum*, lect. 1).

Et ideo rationabiliter videtur sententia Alexandri, quod subjectum huius libri sit ipsum universum, quod dicitur *coelum vel mundus*; et quod de simplicibus corporibus determinatur in hoc libro, secundum quod sunt partes universi. Constituitur autem universum corporeum ex suis partibus secundum ordinem situs: et ideo de illis solum partibus universi determinatur in hoc libro, quae primo et per se habent situm in universo, scilicet de corporibus simplicibus. Et ideo de quattuor elementis non determinatur in hoc libro secundum quod sunt calida vel frigida, vel aliquid huiusmodi: sed solum secundum gravitatem et levitatem, ex quibus determinatur eis situm in universo. . . . Et hoc consonat ei quod consuevit apud Latinos dici, quod in hoc libro agitur *de corpore mobili ad situm*, sive *secundum locum*: qui quidem motus communis est omnibus partibus universi (*De Coelo et Mundo*, lib. 1, proemium).

[113] Cf. footnote 83, *supra*.

[114] J. Maritain, *op. cit.*, p. 81.

at the physical level, lacking the reassurance which mathematical principles impart to physico-mathematical science, must look beyond mathematics to metaphysical principles for the support and defense of their own principles and conclusions.

The philosophy of nature, the science of qualitative being as immutable (*ens mobile ut immutabile*), can alone reveal the ultimate ontological principles underlying motion and change. Physico-mathematical science, "having abandoned the direct search for real causes in order to devote itself to the translation of the measurements of things in a coherent system of equations," [115] reveals only the quantitative connection with qualitative change. But the ultimate explicative value of the philosophy of nature arises from its position at the highest point, in *esse scibili*, of all the sciences in the physical order. For even in the physical order, at the lowest level on intelligibility, metaphysics reigns supreme through its transcendent, immutable principles.

> In the determination of the nature and true value of physico-mathematical science, the place, the part, and the bearing of its explications, metaphysics not only maintains order in the system of our forms of knowledge, but renders to physico-mathematics the essential service of protecting it against otherwise almost inevitable deformations, above all, against the pernicious illusion that it is itself called on to be a philosophy of nature and the belief that things only begin to exist when submitted to the measurement of our instruments.[116]

It is in its clear indication of the inter-relation and respective limits of the basic fields of science, that the principal merit of our diagram is seen. For it is only by a precise determination of the position of mathematics in the hierarchy of speculative science that its regulated and regulative functions can be properly evaluated. The diagram is proposed simply as a preliminary step in that direction, a step toward Maritain's ultimate goal of "systematic interpretation."

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[115] *Ibid.*, p. 56.

[116] *Ibid.*, p. 80.